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Implications of Hypoplastic Compaction Laws on Subsidence Modeling

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The purpose of this note is to address the implications of a reservoir formation exhibiting simple hypoplastic type material behaviour in the analytic and semi-analytic modeling methods that are typically applied to subsidence modeling. This was because a hypoplastic type constitutive law had been suggested as a potentially better description of the experimentally observed compaction behaviour of the Rotliegendes reservoir rock formation. Namely that the rock seemed to exhibit a mixed proportion of both reversible and irreversible volume strain during pore-pressure depletion rather than a distinct elastic-plastic transition at a yield envelope. The regulator (Staatstoezicht Op De Mijnen) queried whether adoption of such a constitutive law for the reservoir compaction would have a significant impact on the subsidence modeling methods.

Hypoplastic constitutive models, strictly speaking, are all those defined by the stress-strain relationship

$$\frac{\partial \sigma}{\partial t} = \mathbf{h}\left(\sigma, \frac{\partial \mathbf{e}}{\partial t}\right) \tag{1}$$

[Kolymbas, 1999] where σ is the stress tensor, $\mathbf{e}$ is the strain tensor, t is time and $\mathbf{h}$ is a tensor function. For the purposes of this note though, consideration is given to all constitutive laws where all volume strains above some small limit (the limit is required to stop the artifact of 'ratcheting') contain a component of irreversible/plastic as well as reversible/elastic volume strain, the proportion increasing monotonically. Small volume strains and all deviatoric strains are assumed to be linear elastic (this latter assumption has a limited range of applicability, as clearly at some scale, large deviatoric strains will become non-linear or irreversible).

Most of the present (semi-)analytic methods used to model and predict surface subsidence due to reservoir pressure depletion, are generally based on the same methodology that Geertsma outlined in his seminal papers on the subject [Geertsma, 1973a; Geertsma, 1973b]. In these, the subsurface is represented by a homogeneous, isotropic, linear elastic [HILE] halfspace and the displacements are calculated using a Green's function for a 'centre of compression nucleus of strain' within such a domain. This Green's function is then convolved with a spatial function that defines the varying strength of the contraction within the reservoir (typically in the form of a pore pressure change field and a spatially dependent ratio that defines the magnitude of the contraction per unit pore pressure change). The theory posits that a pore fluid pressure change in the reservoir rock induces an isotropic stress free strain, (also called a transformation strain or eigenstrain), which is the strain that would occur if the sub-volume concerned was unconstrained by its surroundings, but with no corresponding change in stress (i.e. keeping its in-situ stress state constant). Given as

$$\mathbf{e}^{(u)} = C_{bp} \Delta P_p \mathbf{I} \tag{2}$$

where $\mathbf{e}^{(u)}$ is the stress free strain; C_{bp} is the bulk compressibility with respect to pore pressure [Zimmerman, 1991]; ΔP_p is the change in pore pressure; and I is the unit tensor.

The Green's function for a 'centre of compression nucleus of strain' in a HILE half-space was derived independently by Mindlin & Cheng and Sen [Mindlin & Cheng, 1950; Sen, 1951], where the nucleus is defined as an isotropic set of contracting body forces acting at a single point in the half-space domain (equivalent to three orthogonal force doubles). The magnitude of the body forces, $\mathbf{b}$ for such a centre of compression are determined from the isotropic stress free volume strain at the point scaled by the bulk modulus, K.

$$\mathbf{b} = -K \nabla \mathbf{e}^{(u)} \tag{3}$$

Note that this definition gives rise to some common subtle misconceptions, for while it follows that an isotropic point source of contracting body forces will produce a spherically symmetric strain field in a HILE full-space - that will not be the case for a HILE half space, where the free surface introduces an asymmetry; it is by definition a nucleus of forces, not of in-situ strain. [N.B. while a centre of compression in a HILE full-space will produce a spherically symmetric strain field, in general the superposed strain fields of a distribution of centres of compression will not be so, unless the distribution itself is spherically symmetric] Furthermore, while there is a simple relationship between the (isotropic) stress free volume strain and the in-situ volume strain for any distribution of centre of compression nuclei in a HILE full-space, this is not the case for a HILE half-space. In a HILE half-space in-situ volume strain becomes dependent on the distribution of centre of compression nuclei and on position.

The relationship between isotropic stress free volume strain, $\mathbf{e}^{(u)} = (\varepsilon^{(u)}/3)\mathbf{I}$ (where $\varepsilon^{(u)}$ is a scalar field) and the in-situ volume strain for any distribution of centre of compression nuclei in a HILE full-space is given by

$$\mathrm{tr}(\mathbf{e}^{(f)}) = \frac{1+\nu}{3(1-\nu)}\mathrm{tr}(\mathbf{e}^{(u)}) \tag{4}$$

where $\mathbf{e}^{(f)}$ is the in-situ strain tensor field for a full space. For a HILE material the following identity is true

$$\frac{1+\nu}{3(1-\nu)} = \frac{K}{M} \tag{5}$$

where M is the p-wave or uniaxial strain modulus (in general any elastic modulus can be expressed as a function of two other moduli for a HILE material). Hence if the stress free isotropic volume strain is considered to be related to some effective bulk modulus then the actual resulting in-situ volume strain is scaled to an equivalent effective uniaxial strain modulus. However, this scaling relationship, of bulk to uniaxial modulus, applies to the magnitude of the volume strain, not to it's actual form, i.e. the in-situ strain will not generally be uniaxial. Indeed, for a spherical inclusion undergoing a uniform eigenstrain in a HILE full-space, the resulting constrained strain in the inclusion will also be isotropic, i.e. $\mathbf{e}^{(f)} = [\mathrm{tr}(\mathbf{e}^{(f)})/3]\,\mathbf{I}$.

For the end member case though, of a high aspect ratio, oblate spheroid iclusion, undergoing a uniform eigenstrain (or any thin, laminar type body), in a HILE full-space, the in-situ strain closely approximates to uniaxial strain, normal to the plane (aspect ratios $\gtrsim 10$) [Eshelby, 1957; Segall & Fitzgerald, 1998]. This is a reasonable approximation for many subsurface reservoirs where formation thicknesses are of order 100 m or so, while lateral extents are on the order of kilometers. Hence, on this initial analysis it seems that the appropriate measure to calibrate the subsidence models would be the volume strain due to pore pressure change under uniaxial strain conditions, (working under the maxim that the calibration should be as close to actual conditions as possible to reduce errors and biases). However, there is a minor falacy in this reasoning, in that it conflates the constrained strain in a HILE full-space with that in a HILE half-space.

The Green's function that is at the heart of these models simply relates an isotropic stress free strain to a surface displacement function for a HILE half-space. The only elastic parameter that governs this mapping is Poisson's ratio, the material stiffness, bulk or shear (or variations thereof) is unimportant and cancels out. Similarly, the relationship between the stress free volume strain and the 'effective constrained volume strain if the domain were a HILE full-space' (the measure prefered by Geertsma and used to calibrate models), is also only dependent on Poisson's ratio. The relationship between stress free volume strain and the in-situ constrained strain for a HILE half-space is somewhat more complex and depends on the Poisson's ratio, position and the distribution of the strain. The validity of these relationships though, relies on the material having HILE constitutive properties.

However, the material behaviour governing the magnitude of the stress free strain is completely independent of the constitutive properties of the domain, it is simply imposed a-priori. Hence the process that generates the transformation strain can be anything at all and does not have to obey any particular constitutive law. This is made somewhat more apparent by noting the relative equivalences of Eshelby's models, where a compressive/dilatational nucleus of strain is superposed over an ellipsoid inclusion in a full space domain [Eshelby, 1957], and Geertsma's models where the nucleus of strain is superposed over an inclusion in a half space domain [Geertsma, 1973a; Geertsma, 1973b]. A detailed analysis of the equivalence between these two modeling approaches has been given by Rudnicki [2002; 2011].

It is possible therefore, to consider a scenario where the transformation strain of the reservoir formation occurs hypoplastically and all further strains occur elastically. For reasonable values of Poisson's ratio (0.1 - 0.3) though, this seems implausible, as the strains imposed between the stress free state and the constrained strain state are of the same order as the transformation strain. We can equally consider an alternative scenario where hypoplastic deformation of the reservoir occurs that precisely matches the constrained strain that would occur for the half-space being considered. Again though this is implausible as the constrained strain state for a half space is a complex, spatially varying function, dependent on the shape of the reservoir itself. However, an approximate hybrid approach suggests itself, where the reservoir is deformed hypoplastically to a true or permitted constrained strain state that is close to that which would occur in-situ, and that further perturbations to match the constraint and boundary conditions incur only elastic deformations. While this can only approximate the true strain state it can be reasonably accurate.

To analyse efficacy of this hybrid approximate method, we consider the Green's function of the in-situ volume strain for a centre of compression at the point $\mathbf{y} = (y_1, y_2, y_3)$

$$\mathrm{tr}(\mathbf{e}^*(\mathbf{x} = \mathbf{y}; \mathbf{y}) = \frac{1+\nu}{3(1-\nu)}\bar{\varepsilon}\left[\delta(\mathbf{x}-\mathbf{y}) + \frac{1-2\nu}{4\pi\|\mathbf{x}-\mathbf{y}'\|^3}\right] \tag{6}$$

It is apparent from this that for reservoir bodies that are approximately as deep as they are laterally extensive, or deeper, that the in situ volume strain approaches that for a HILE full-space within <5 stated, for a reservoir inclusion with a lateral extent to thickness ratio of 10 or more, that the uniaxial strain state is a true constrained strain state for a HILE full-space.

The conclusion is that for thin, laterally extensive reservoir formations (extent to thickness ratio $\gtrsim 10$), that are about as deep as they extend laterally, that an accurate solution for the surface subsidence can be found by applying the standard methodology, as corrections for hypoplastic behaviour will be small. This is because the uniaxial strain measurement is essentially the true constrained state that such a reservoir would experience in-situ. The reservoir types described are a reasonably close match to the gas field reservoirs of the Waddenzee area. For reservoirs that diverge from these types, i.e. thick reservoir sections or lying at proportionately shallow depth, then elastic correction terms will need to be applied and accuracy will deteriorate.

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